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An Improved Geometric Approximation for the Beta Geometric Distribution

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Abstract

The paper gives an approximation of the beta geometric distribution with parameters α and β by an improved geometric distribution with parameter $\frac{\alpha}{\alpha+\beta}$. The improved geometric approximation is more accurate than the geometric approximation when α is large.

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1 Introduction

The beta geometric distribution with parameters $\alpha > 0$ and $\beta > 0$ is the geometric distribution with a random variable p that has a beta distribution with shape parameters $\alpha > 0$ and $\beta > 0$. Let X be this beta geometric random variable with the probability function as follows:

$$\mathbf{bg}_{\alpha,\beta}(x) = \frac{\alpha\Gamma(\beta+x)\Gamma(\alpha+\beta)}{\Gamma(\beta)\Gamma(\alpha+\beta+x+1)},\ x = 0, 1, ..., \tag{1.1}$$

and has mean $\mu = \frac{\beta}{\alpha-1}$ and variance $\sigma^2 = \frac{\alpha\beta(\alpha+\beta-1)}{(\alpha-2)(\alpha-1)^2}$. It is observed that $\alpha \to \infty$ while $q = 1-p = \frac{\beta}{\alpha+\beta}$ remains a constant, then $\mathbf{bg}_{\alpha,\beta}(x) \to \mathbf{g}_p(x) = q^x p$ for every $x \in \mathbb{N}\cup\{0\}$. In this case, [2] gave a bound for the total variation distance between the beta geometric and geometric distributions. In this study, we are interest to determine an improved geometric probability function, $\widehat{\mathbf{g}}_p(x)$, to approximate the beta geometric probability function in (1.1). The accuracy of the approximation is measured in the form of $\left|\mathbf{bg}_{\alpha,\beta}(x) - \widehat{\mathbf{g}}_p(x)\right|$ for $x \in \mathbb{N} \cup \{0\}$, which is in Section 2. In Section 3, some numerical examples have been given to illustrate the improved approximation and the conclusion of this study is presented in the last section.

2 Result

The following lemma is directly obtained from the same idea in [1].

Lemma 2.1. *For $\alpha > 2$, $\beta > 0$, $x \in \mathbb{N}$ and $0 < q < 1$, then*

$$\prod_{i=0}^{x-1}\left(q + \frac{i}{\alpha+\beta}\right) = q^x\left\{1 + \frac{x(x-1)}{2(\alpha+\beta)q} + O\left(\frac{1}{\alpha^2}\right)\right\}, \tag{2.1}$$

$$\prod_{i=1}^{x}\left(1 + \frac{i}{\beta+n}\right) = 1 + \frac{x(x+1)}{2(\alpha+\beta)} + O\left(\frac{1}{\alpha^2}\right). \tag{2.2}$$

Theorem 2.1. *For $x \in \mathbb{N} \cup \{0\}$ and $p = \frac{\alpha}{\alpha+\beta}$, we have*

$$\mathbf{bg}_{\alpha,\beta}(x) = \widehat{\mathbf{g}}_p(x) + O\left(\frac{1}{\alpha^2}\right), \tag{2.3}$$

$$\mathbf{bg}_{\alpha,\beta}(x) \approx \widehat{\mathbf{g}}_p(x) \tag{2.4}$$

for large α, where $\widehat{\mathbf{g}}_p(x) = \frac{q^x p\left\{1+\frac{x(x-1)}{2\beta}\right\}}{1+\frac{x(x+1)}{2(\alpha+\beta)}}$.

Proof. Applying Lemma 2.1, it follows that

$$\begin{aligned}\mathbf{bg}_{\alpha,\beta}(x) &= \frac{\alpha[\beta\cdots(\beta+x-1)]}{(\alpha+\beta)\cdots(\alpha+\beta+x)}\\ &= \frac{\prod_{i=0}^{x-1}\left(q+\frac{i}{\alpha+\beta}\right)p}{\prod_{i=1}^{x}\left(1+\frac{i}{\alpha+\beta}\right)}\\ &= \frac{q^x p\left\{1+\frac{x(x-1)}{2\beta}\right\}}{1+\frac{x(x+1)}{2(\alpha+\beta)}} + O\left(\frac{1}{\alpha^2}\right)\\ &= \widehat{\mathbf{g}}_p(x) + O\left(\frac{1}{\alpha^2}\right).\end{aligned}$$

If α is large, then $O\left(\frac{1}{\alpha^2}\right) \approx 0$. Hence $\mathbf{bg}_{\alpha,\beta}(x) \approx \widehat{\mathbf{g}}_p(x)$. □

3 Numerical examples

The following examples are given to illustrate how well an improved geometric distribution approximates a beta geometric distribution.

3.1. Let $\alpha = 90$ and $\beta = 10$, then $p = \frac{90}{100}$ and the numerical results are as follows:

x	$\mathbf{bb}_{n,\beta}(x)$	$\widehat{\mathbf{g}}_p(x)$	$\mathbf{g}_p(x)$	$\lvert\mathbf{bb}_{n,\beta}(x) - \widehat{\mathbf{g}}_p(x)\rvert$	$\lvert\mathbf{bb}_{n,\beta}(x) - \mathbf{g}_p(x)\rvert$
0	0.90000000	0.90000000	0.90000000	0.00000000	0.00000000
1	0.08910891	0.08910891	0.09000000	0.00000000	0.00089109
2	0.00960978	0.00961165	0.00900000	0.00000187	0.00060978
3	0.00111959	0.00110377	0.00090000	0.00001581	0.00021959
4	0.00013995	0.00013091	0.00009000	0.00000904	0.00004995
5	0.00001866	0.00001565	0.00000900	0.00000301	0.00000966
6	0.00000264	0.00000186	0.00000090	0.00000078	0.00000174
7	0.00000039	0.00000022	0.00000009	0.00000018	0.00000030
8	0.00000006	0.00000003	0.00000001	0.00000004	0.00000005
9	0.00000001	0.00000000	0.00000000	0.00000001	0.00000001

3.2. Let $n = 30$ and $\beta = 110$, then $p = \frac{110}{140}$ and the numerical results are as follows:

x	$\mathbf{bb}_{n,\beta}(x)$	$\widehat{\mathbf{g}}_p(x)$	$\mathbf{g}_p(x)$	$\lvert\mathbf{bb}_{n,\beta}(x) - \widehat{\mathbf{g}}_p(x)\rvert$	$\lvert\mathbf{bb}_{n,\beta}(x) - \mathbf{g}_p(x)\rvert$
0	0.76923077	0.76923077	0.76923077	0.00000000	0.00000000
1	0.17615972	0.17615972	0.17751479	0.00000000	0.00135507
2	0.04137084	0.04137563	0.04096495	0.00000479	0.00040589
3	0.00995389	0.00994003	0.00945345	0.00001386	0.00050044
4	0.00245133	0.00243089	0.00218157	0.00002044	0.00026976
5	0.00061737	0.00060181	0.00050344	0.00001556	0.00011393
6	0.00015888	0.00015003	0.00011618	0.00000885	0.00004270
7	0.00004175	0.00003750	0.00002681	0.00000425	0.00001494
8	0.00001119	0.00000937	0.00000619	0.00000183	0.00000501
9	0.00000306	0.00000233	0.00000143	0.00000073	0.00000163
10	0.00000085	0.00000058	0.00000033	0.00000027	0.00000052
11	0.00000024	0.00000014	0.00000008	0.00000010	0.00000017
12	0.00000007	0.00000004	0.00000002	0.00000003	0.00000005
13	0.00000002	0.00000001	0.00000000	0.00000001	0.00000002
14	0.00000001	0.00000000	0.00000000	0.00000000	0.00000001

From the examples 3.1 and 3.2, it can be seen that the improved geometric approximation is more accurate than the geometric approximation.

4 Conclusion

In this study, an improved geometric approximation for the beta geometric distribution with parameters α and β was derived. By numerical comparison, the improved geometric approximation is more accurate than the geometric approximation when α is sufficiently large, that is, the beta geometric distribution with parameters α and β can be well approximated by an improved geometric distribution with parameter $\frac{\alpha}{\alpha+\beta}$ when α is sufficiently large.

References

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